Title: At The Circus

Brief Overview:

Students will investigate a real world problem using technology. They will use Geometer's Sketchpad to construct and measure segments and to discover relationships between various lengths. The unit assessment will reflect an understanding of the concept of ratios and systems of linear equations. Several alternative methods for solving the problem will be developed.

NCTM 2000 Principles for School Mathematics:

- **Equity:** *Excellence in mathematics education requires equity - high expectations and strong support for all students.*
- **Curriculum:** *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.*
- **Teaching:** *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*
- **Learning:** *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*
- **Assessment:** *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*
- **Technology:** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.*

Links to NCTM 2000 Standards:

- **Content Standards**

Number and Operations
Students will be able to understand the relationships between various segment lengths.

Algebra
Students will be able to represent and analyze mathematical situations by using system of equations.

Geometry
The students will be able to analyze characteristics and properties of two-dimensional geometric shapes and develop mathematical arguments about geometric relationships. They will be able to specify locations and describe spatial relationships using coordinate geometry and constructing a physical model.

Measurement

The students will be able to understand measurable attributes of objects and the units, systems and the process of measurement. They will be able to apply appropriate techniques and tools to determine measurements.

- **Process Standards**

Mathematics as Problem Solving, Reasoning and Proof, Communication, Connections and Representation

These five process standards are threads that integrate throughout the unit, although they may not be specifically addressed in the unit. They emphasize the need to help students develop the processes that are the major means for doing mathematics, thinking about mathematics, understanding mathematics, and communicating mathematics.

Students will use Geometer's Sketchpad to construct a model of the problem situation and propose solutions to a problem related to the circus. They should be able to express in written form the problem using mathematical terminology specifically related to the problem. There should be group efforts displayed in the classroom environment. Students should be able to model and construct information that is pertinent to the class assignment. There should be a display of understanding through modeling that generates feedback within the classroom.

Links to Maryland High School Mathematics Core Learning Units:

Geometry, Measurement, and Reasoning

- **2.1.1**
  The student will analyze the properties of geometric figures and will construct geometric figures using technology.

- **2.1.2**
  The student will identify and/or verify properties of geometric figures using the coordinate plane and concepts from algebra.

- **2.1.4**
  The students will construct and/or draw and/or validate properties of geometric figures using appropriate tools and technology.

- **2.2.2**
  The students will solve problems using two-dimensional figures.

- **2.3.2**
  The student will use techniques of measurement and will calculate various lengths.

Grade/Level:

Grades 9 – 12, Geometry

Duration/Length:

Two to three class periods, approximately 45-50 minutes in length

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Using Geometer's Sketchpad to draw basic geometric figures
- Programming on the TI-83 plus (optional)
- Graphing linear equations on the TI-83 plus
- Writing equations of lines
- Solving a pair of linear equations

Student Outcomes:

Students will:

- Be able to model a real world problem in a coordinate framework.
- Be able to apply coordinate geometry.
- Be able to apply the concept of ratios to formulate a conjecture.
- Be able to test a conjecture using Geometer's Sketchpad and modify the conjecture as needed.

Materials/Resources/Printed Materials:

- Textbook
- Computer with Geometer's Sketchpad (or TI-92 plus)
- TI-83 plus
- Student activity sheets, homework sheets, and assessment sheets
- Push-pins, meter sticks/yard sticks, tape and strings

Development/Procedures:

(1) Teacher will review any necessary computer/calculator skills and procedures. (Worksheets and Assessment were developed using Geometer's Sketchpad 3.0.)

(2) <u>Day 1:</u>
Teacher should lead students in a discussion of the problem and present a physical model of the problem situation.
They should also give explanations about applying Geometer's Sketchpad to this problem.

(3) <u>Day 2:</u>
Students will complete Worksheet 1 using Geometer's Sketchpad.

The optional extra credit (Worksheet 2) can be assigned for homework.

(4) Day 3:
For assessment purposes, students will complete Worksheet 3.

Assessment:

This is an activity-based unit with several worksheets students will complete and have checked. They will also use the TI-83 plus to execute a program to verify their conjectures. A formal assessment will be given at the end of the unit.

Extension/Follow Up:

The students can vary the height of the poles and examine the relationship between these heights and the point of intersection of the ropes, while varying the distance between the poles. They can perform this exercise using Geometer's Sketchpad and/or the TI-83 plus.

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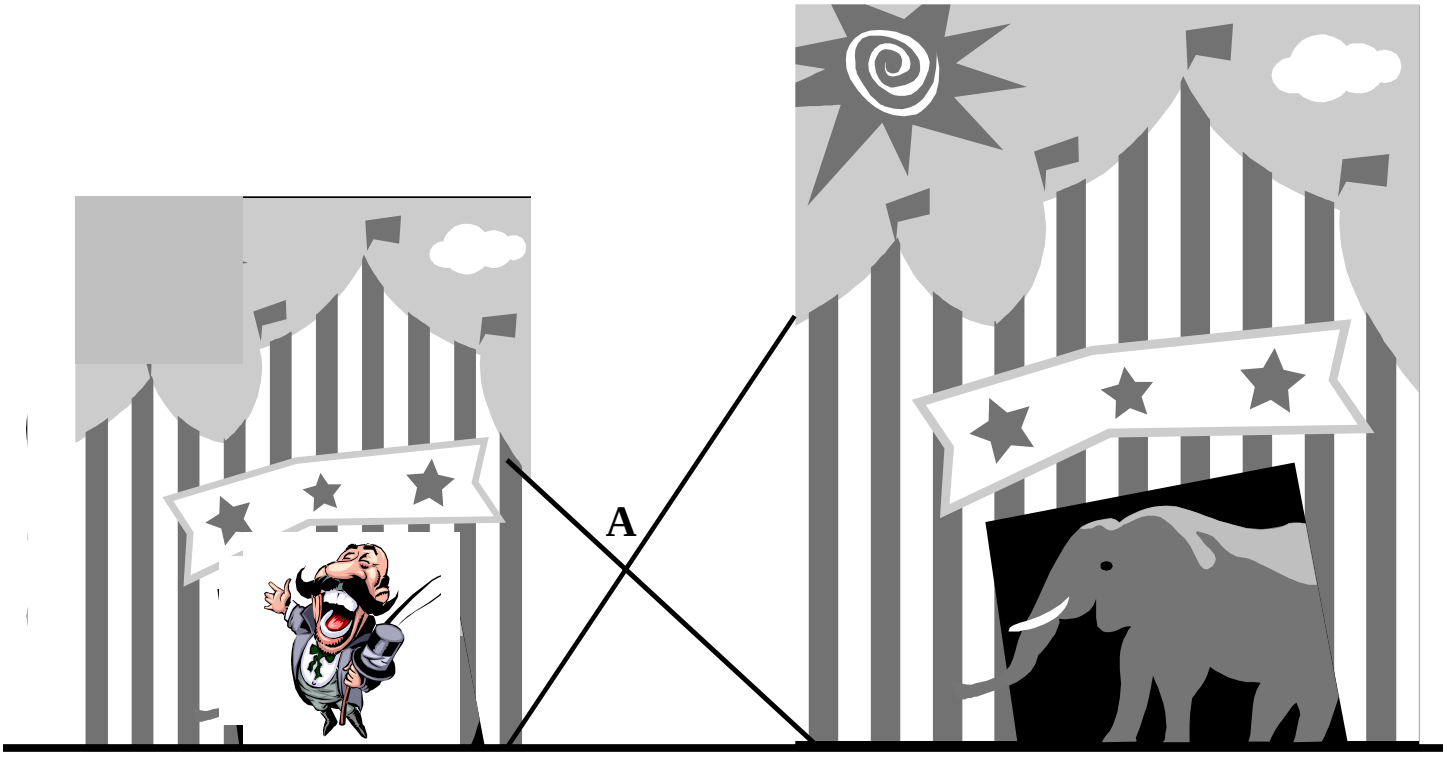
AT THE CIRCUS

A Technology-Driven Geometry Investigation



PICTURE

Circuses erect tents of different sizes in which the acrobats, clowns, and animal acts perform. The tents have poles at each corner held vertical by ropes attached to the ground. The diagram above shows two adjacent tents. The ropes holding up the poles closest to each other are tied so that the top of each pole is attached to the bottom of the other. The ropes cross somewhere in between the tents, which is a main pathway for people visiting the circus. The ropes must cross at a point high enough above the ground (point A in the PICTURE) to allow people to pass under it without ducking. One of the poles is 20 feet tall and the other is 30 feet tall.

Is there enough information to determine the height above the ground of the point at which the ropes cross? If so, what is the height? Will it be high enough for an adult of average height to pass under without ducking?

In order to address the problem, it is important to first examine the following question:

Is it necessary to know the distance between the poles? That is will the height of point A in the PICTURE vary, depending upon how far apart the two poles are placed?

A GUIDE FOR THE TEACHER

1. Distribute copies of “At the Circus” to students. Have students read the handout and discuss in pairs the questions posed in the reading.

2. Construct a physical model

   Attach push pins near the bottom and at the 20-inch mark of one yard/meter stick and near the bottom and at the 30-inch mark of a second yard/meter stick. Stand both sticks upright on the ledge of a chalkboard (leaning slightly against the board), 4 to 5 feet apart. Using two pieces of string, each about 8 to 10 feet long, wrap the ropes around the push pins to create a model resembling the PICTURE on the student’s handout. (You may want to have student volunteers helping.) Place a mark on the chalkboard at the spot where the ropes intersect.

   Repeat this process at least two more times with the meter sticks placed further apart, then closer together. Place marks on the board at the spots where the ropes cross in each case.

   Students should observe that all the marks on the chalkboard are approximately collinear and approximately the same height above the ledge of the board (the line through them is approximately horizontal).

3. Simulating using Geometer's Sketchpad (Worksheet 1)

   In worksheet 1, students will use Geometer's Sketchpad so that pulling the "poles" apart, or pushing them together, reveals that the height of the point at which the ropes cross remains constant. Also, if the poles are made to measure 2 inches and 3 inches (rather than 20 and 30 feet) on the computer, then the height of the point at which the ropes cross will appear as 1.2. Students can be led to realize that this is equivalent to 6/5, (600/50 using the measurements as originally given) which should suggest a general answer. Since 600/50 = 12, there is 12 feet of clearance under the ropes, more than enough for a person of average height to easily walk without ducking.

4. Simulating graphically (the assessment of “At the Circus”)

   Students will make a graphic representation on paper, using the origin as the bottom of the first pole and (0, 20) as the top of the first pole. By representing the bottom and top of the second pole with (d, 0) and (d, 30), respectively, and replacing the variable d with a convenient value (representing the distance between the poles), they can compute the equations of the lines representing the ropes, and algebraically solve for the point of

intersection. The *y*-coordinate of this point is the height of the intersection of the ropes. By changing the value of *d* and repeating the process, they can verify that the *y*-coordinate is unchanged.

5. Simulating graphically using the TI-83 plus graphing calculator (Worksheet 2: Extra Credit)

   The program "POLES" in Worksheet 2 will graph a simulation of the PICTURE on the TI-83 plus. This program may be linked from the teacher's calculator for students who would have a difficult time programming the calculator themselves. It will allow students to use any heights for the poles and any distance between the poles. It will record, in Y_1 and Y_2 menus, the equations of the lines representing the ropes, and set an appropriate window for the diagram.

   Students can use the intersect option in the 2^{nd} trace menu to identify the point of intersection of the ropes (lines). The *y*-coordinate of this point represents the height of the intersection point of the ropes.

   Executing the program again, changing the distance between the poles while keeping the heights of the poles the same, should verify that the height of the point of intersection of the ropes is independent of the distance between the poles. The students should observe that the final answer is the ratio of the product of the heights of the poles and the sum of the heights.

Name_____

Date_____

Worksheet #1: Modeling with Geometer's Sketchpad

1. Draw a horizontal line.
2. Choose two points on the line that are not too close together. Label them D and E. (Do not use the points that are constructed with the line).
3. Measure the distance between D and E.
4. With D and E highlighted and the line highlighted, construct lines perpendicular to line DE at D and E. Both perpendiculars should be above line DE.
5. Mark a point on the perpendicular above D and label it B. Similarly, mark a point above E and label it C. Your diagram should begin to resemble Fig.1 below.

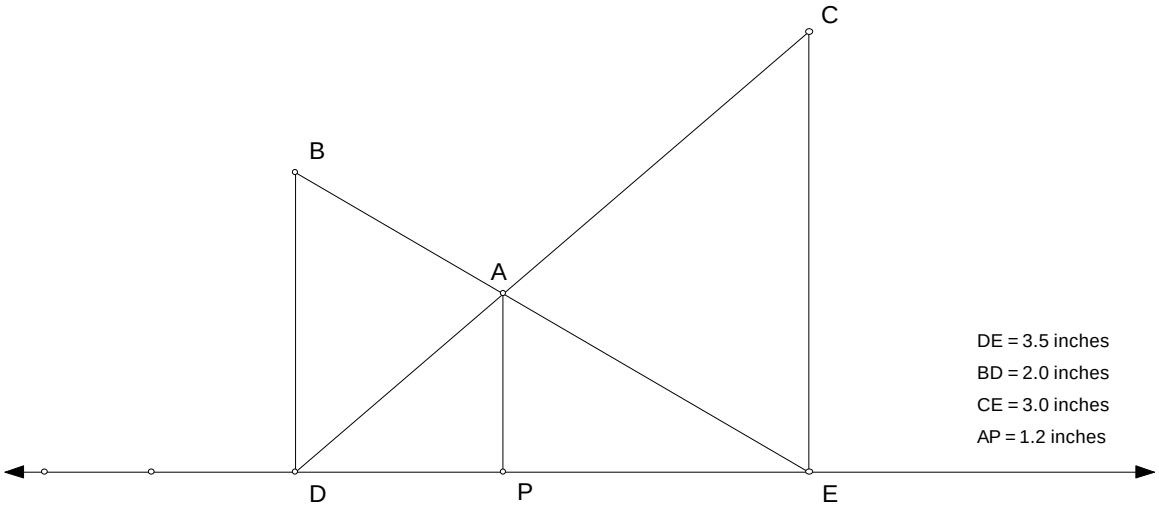


Fig.1

6. Measure segments BD and CE.
7. Highlight B and drag it until segment BD is 2 inches in length. Similarly, highlight C and drag it until segment CE is 3 inches in length.
8. Go the tool box and change the line symbol to segment.
9. Construct segments BD and CE.
10. Click on line BD above point B, go to Display and click on Hide Line. Similarly, click on line CE above C, go to Display and click on Hide Line.
11. Construct segments BE and CD.
12. Highlight segments BE and CD, construct the point of intersection and label it A.
13. Highlight A and line DE and construct a perpendicular line through A.
14. Highlight the perpendicular line through A and line DE, construct the point of intersection and label it P.
15. Construct segment AP.

16. Highlight line AP above point P, go to Display and click on Hide Line.
17. Measure segment AP.

18. Record the measure of the following segments:

    AP= _____ DE=_____

19. Highlight point E and drag it to the right to widen the distance from D to E.
20. Record the measure of the following segments:

    AP=_____ DE=_____

21. Repeat step 19 and record a third set of measures:

    AP=_____ DE=_____

22. What conjecture can you make about the measure of AP?

    _____

    _____

    _____

23. In the original problem (with the poles 20 and 30 feet tall) how many feet above the ground do the ropes cross? _____

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EXEC EDIT NEW
1:POLES
```

```
PROGRAM:POLES
:ClrDraw
:ClrHome
:Disp "HEIGHT OF
 LEFT POLE?"
:Input A
:Disp "HEIGHT OF
 RIGHT POLE?"
```

```
PROGRAM:POLES
:Input B
:Disp "DISTANCE
BETWEEN POLES?"
:Input D
:{0,D}→L1
:{0,B}→L2
:0→Xmin
```

```
PROGRAM:POLES
:D→Xmax
:-1→Ymin
:B+1→Ymax
:LinReg(ax+b) L1
,L2,Y1
:{0,D}→L1
:{A,0}→L2
```

```
PROGRAM:POLES
:LinReg(ax+b) L1
,L2,Y2
:Line(0,0,0,A)
:Line(D,0,D,B)
:Line(0,0,D,0)
:
:
```

WORKSHEET #2: Extra Credit

The program "POLES" will graph a simulation of Fig.1. Execute the program and record the lengths of segments BD, CE, DE and AP.

Step 1:
Length BD = 2 inches
Length CE = 3 inches
Length DE = 10 inches
Length AP _____

Step 2:
Change the distance between the two poles **only**, and record the lengths of segments DE and AP.

Length BD = 2 inches
Length CE = 3 inches
Length DE _____
Length AP _____
Rewrite length AP as a fraction _____

What happened to the length of segment AP? _____
Does your conclusion support the result from the activity using the Geometer's Sketchpad? _____

Step 3:
Now change the length of the poles while varying the distance between the poles. Record the lengths of the segments BD, CE, DE and AP.

Length BD _____
Length CE _____
Length DE _____
Length AP _____
Rewrite length AP as a fraction _____

Step 4:
Repeat Step 3.

Length BD _____
Length CE _____
Length DE _____
Length AP _____
Rewrite length AP as a fraction _____

Is there a relationship between AP as a fraction and the lengths of BD and CE?
_____
_____
_____

Name_____
Date_____

WORKSHEET # 3: AT THE CIRCUS ASSESSMENT

In this exercise you will simulate graphically the situation represented in Fig.1.

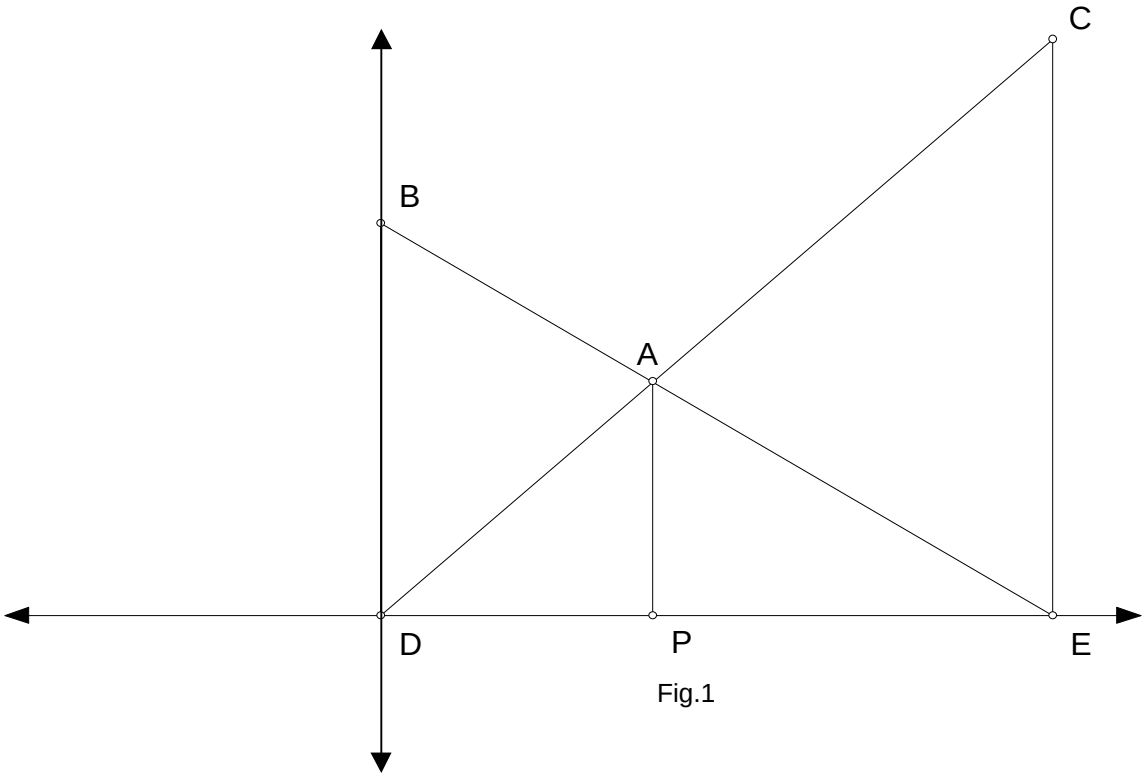

Fig.1

(1) Using a graph paper, plot (0, 0) and (0, 20). Let these points represent the bottom and top of the first pole.
(2) Pick a whole number between 1 and 10 and call this number d. Let (*d*, 0) and (*d*, 30) represent the bottom and top of the second pole.
(3) Plot the coordinates for the second pole on the graph paper.

Now answer the following questions:

a) What value did you choose for *d*? (1 point)_____

b) What is the distance between the two poles? (2 points) _____

Using your coordinates, write an equation to represent each rope that is used to hold the two poles vertical. (Segments BE and CD in Fig.1)

c) Equation of BE (2 points) _____

d) Equation of CD (2 points) _____

e) Explain how you calculate your equations? (3 points) _____
_____
_____

Now put your equations in the graphing calculator and calculate the coordinates of point A.

f) Name the coordinates of A. (2 points) _____

g) What key strokes did you use to find the coordinates of A? (Accuracy of the method chosen is important) (3 points) _____
_____

h) What is the length of segment AP? (2 points) _____

i) Explain your reasoning for your conclusion in part (h) (3 points) _____
_____

Take a different value for *d* and record the following information:

j) What value did you choose for *d*? (1 point) _____

k) What is the distance between the two poles? (2 points) _____

Using your new coordinates, write the equations for:

l) segment BE (2 points) _____

m) segment CD (2points) _____

n) Name the coordinates of A. (2 points) _____

o) What is the length of segment AP? (2 points) _____

p) What happened to the length of segment AP? (2 points) _____

WORKSHEET #3: AT THE CIRCUS ASSESSMENT ANSWER KEY

a) 5, for example (any number between 1 and 10)
b) 5 (must be the same number as part a) above)
c) If $d = 5$, then the slope of segment *BE* is 20/-5 or –4, $y = -4x + 20$
d) If $d = 5$, then the slope of segment *CD* is 30/5 or 6, $y = 6x + 0$
e) Count up 20, move left 5, write 20/-5, then divide to produce –4; and count up 30, move right 5, write 30/5, then divide to produce 6 or use the coordinates (0, 20) to (5, 0), then the slope formula: (0- 20)/ (5- 0) = -20/5 = -4 and (0, 0) to (5, 30), using the formula: (30- 0)/ (5- 0) = 30/5 = 6. Now look for the *y*- intercept, the point where segment *BE* crosses the *y*- axis, at the 20. Similarly, segment *CD* crosses the *y*- axis at 0.
f) (2, 12)
g) Press *calc*, then #5, then hit *enter* three times.
h) 12
i) The vertical distance from the *x*- axis to (2, 12) is 12 spaces.
j) 8, for example, (any number between 1 and 10, but 5, the number chosen in part a)
k) 8 (The answer here must match the answer in j.)
l) If $d = 8$, the slope of segment *BE* is 20/-8 or 5/-2. The *y*- intercept is 20. The equation of the segment is $y = (5/-2)x + 20$.
m) If $d = 8$, the slope of segment *CD* is 30/8 or 15/4. The *y*- intercept is 0. The equation of the segment is $y = (15/4)x + 0$.
n) (32, 12)
o) $AP = 12$
p) *AP* stayed the same.

RUBRIC FOR WORKSHEET # 3: AT THE CIRCUS ASSESSMENT

Question a) 1 point: A student selects a number between 1 and 10.
0 points: A student selects a number other than between 1 and 10.

b) 2 points: The student's selection must match the number chosen in part a.
0 points: The student's selection does not match.

c) and d) 2 points: A student writes the correct equation in slope-intercept form.
1 point: A student writes an equation in slope-intercept form with the correct slope and the incorrect *y*-intercept. Or the student writes an equation in slope-intercept form with an incorrect slope and correct *y*-intercept.
0 points: A student writes an equation with an incorrect slope and incorrect *y*-intercept.

e) 3 points: A student writes the correct explanation for the slope and *y*-intercept.
1 point: A student writes an incorrect method of calculating the slope or *y*-intercept.
0 points: A student writes the incorrect method of calculating both slope and *y*-intercept.

f) 2 points: A student has both coordinates correct.
1 point: A student has only one coordinate correct.
0 points: A student has neither coordinate correct.

g) 3 points: A student lists the correct key strokes.
2 points: A student uses the trace button to approximate the intersection.
0 points: A student describes an incorrect method.

h) 2 points: A student writes the correct length based on the answer he put in part f).
0 points: A student writes the incorrect length.

i) 3 points: A student writes the correct explanation.
0 points: A student writes an incorrect explanation.

j) 1 point: A student chooses a number between 1 and 10 that is different from the number he chose in part a).
0 point: A student picks any other number.

k) 2 points: A student's number must match the answer he chose in part j).
0 points: A student's number does not match part j).

l) and m) These two questions should be scored the same as parts c) and d) above.

n) This question should be scored the same as question f) above.

o) This question should be scored the same as question h) above.

p) 2 points: A student states that *AP* remained the same or an answer that correctly identifies the relationship between the student's response in h) and o).
0 points: A student writes an incorrect answer.